CS131 Homework #5 (18 pts, **each item is 3 pts**)

1. Prove that 12 − 22 + 32 −···+ (−1)n−1n2 = (−1)n−1 n(n + 1)/2, whenever n is a positive integer.

Use math. induction.

Base case: n=1 1=(−1)1−1 1\*2/2

Inductive hypothesis: 12 − 22 + 32 −···+ (−1)n−1n2 = (−1)n−1 n(n + 1)/2

Prove the statement for n+1:

12 − 22 + 32 −···+ (−1)n−1n2 +(−1)n(n+1)2 =

= (−1)n−1 n(n + 1)/2 + (−1)n(n+1)2

=(−1)n (-n2- n+2n2 +4n + 2)/2=

=(−1)n (n2+3n + 2)/2=

=(−1)n (n+1)(n + 2)/2.

1. Prove that n! < nn, where n is an integer greater than 1.

Use math. induction.

Base case: n=2: 2!=2< 22=4

Inductive hypothesis: n! < nn

Prove the statement for n+1:

(n+1)! =(n+1)n!< (n+1)nn< (n+1)(n+1)n= (n+1)n+1

1. Prove that 3 divides n3 + 2n whenever n is a positive integer.

Use math. induction.

Base case: 3 divides (1+2)=3

Inductive hypothesis: 3 divides n3 + 2n, so n3 + 2n=3k, for some integer k

Goal: (n+1)3 + 2(n+1) divisible by 3.

Proof: (n+1)3 + 2(n+1)= n3 +3n2 +3n+1+2n+2=( n3 + 2n)+ 3n2 +3n+3=3k+3n2+3n+3=

=3(k+n2+n+1) – divisible by 3

1. Show that n cents payment for n ≥ 8 can be formed using just 3-cent and 5-cent stamps. (Use strong induction.)

Base cases:

n=8: one 3-cent and one 5-cent

n=9: three 3-cents

n=10: two 5-cent

Ind. hyp: statement is true for all 8≤k<n.

Prove the statement for n≥11:

n=(n-3)+3 is possible to form using the same stamps as (n-3) with one more 3-cent stamp. (n-3) could be formed by 3-cent and 5-cent stamps by inductive hypothesis.

1. Find f (2), f (3), f (4), and f (5) if f is defined recursively by f (0) = f (1) = 1

and f (n + 1) = f (n)f (n − 1) for n = 1, 2,...

Answer: f (2)=f (3)=f (4)=f (5)=1

1. Write a recursive definition of the number of leaves in a binary tree.

Base step:

empty tree – leaves(T)=0

one node tree – leaves(T)=1

Recursive step:

2 trees T1 and T2 connected by a root,

number of leaves = leaves (T1)+leaves(T2)

(If students forgot about empty tree base case – add it in the comments, but don’t take off points).